WS#7-2
The Law of Sines

1. **The Law of Sines states** For a triangle w/ sides a, b, c & opposite angles α, β, γ, respectively, $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

2. **Possibilities:**

Case 1: SAA OR ASA (one side & two angles)

Solve the triangle: $\alpha = 35°, \beta = 15°, c = 5$

$\gamma = 180 - (35+15) = \boxed{130°}$ $\quad \frac{\sin 130°}{5} = \frac{\sin 35°}{a}$

Case 2: SSA or the Ambiguous Case (2 sides & the angle opposite one of them)

No triangle	$a < b\sin\alpha$
One triangle	$a = b\sin\alpha$ **or** $a \geq b$
Two triangles	$b\sin\alpha < a < b$

$\frac{\sin 15°}{b} = \frac{\sin 130°}{5}$ $\quad \frac{a\sin 130°}{\sin 130°} = \frac{5\sin 35°}{\sin 130°}$

$b = \frac{5\sin 15°}{\sin 130°}$ $\quad \boxed{a \approx 3.74}$

$\boxed{b \approx 1.69}$

A. Solve the triangle: $a = 3,\ b = 2,\ \alpha = 40°$

* One △ b/c $a \geq b$

① $\frac{\sin 40°}{3} = \frac{\sin\beta}{2}$ $\quad \sin\beta = \frac{2\sin 40°}{3}$

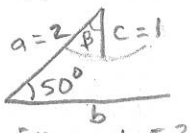
② $\gamma = 180 - (40 + 25.4)$ $\quad \boxed{\gamma = 114.6°}$

③ $\frac{\sin 40°}{3} = \frac{\sin 114.6°}{c}$ $\quad \boxed{c \approx 4.24}$ $\quad \beta = \sin^{-1}\left[\frac{2\sin 40°}{3}\right] \approx \boxed{25.4°}$

B. Solve the triangle: $a = 2,\ c = 1,\ \gamma = 50°$

* No such △

$\frac{\sin\alpha}{2} = \frac{\sin 50°}{1}$



$\sin\alpha = 2\sin 50 \approx 1.53$

• There is NO ANGLE that Allows $\sin\alpha > 1$

C. Solve the triangle: $a = 6,\ b = 8,\ \alpha = 35°$

* Two △'s $b\sin\alpha < a < b$

$8\sin 35° < 6 < 8$

① $\frac{\sin 35°}{6} = \frac{\sin\beta}{8}$ $\quad \beta = \sin^{-1}\left[\frac{8\sin 35°}{6}\right] \approx 49.9°$

$\beta_1 = 49.9°,\ \beta_2 = 180 - 49.9 = 130.1°$

$\gamma_1 = 180 - \alpha - \beta_1 = 180 - 35 - 49.9 \approx 95.1°$

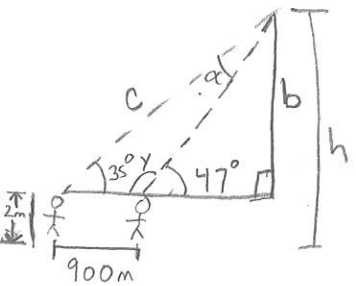
② $\frac{\sin 35}{6} = \frac{\sin 95.1}{c_1}$ $\quad c_1 = \frac{6\sin 95.1}{\sin 35} \approx 10.42$

③ $\frac{\sin 35}{6} = \frac{\sin 14.9}{c_2}$ $\quad c_2 = \frac{6\sin 14.9}{\sin 35}$ $\quad c_2 \approx 2.69$

$\gamma_2 \approx 180 - \alpha - \beta_2$

$\gamma_2 \approx 180 - 35 - 130.1 \approx 14.9°$

3. To measure the height of a mountain, a surveyor takes two sightings of the peak at a distance 900 meters apart on a direct line to the mountain. The first observation results in an angle of elevation of 47° and the second results in an angle of elevation of 35°. If the transit is 2 meters high, what is the height of the mountain?



① $\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$ $\quad \gamma = 180 - 47 = 133°$

$\alpha = 180 - (35 + 133) = 12°$

$\frac{\sin 12°}{900} = \frac{\sin 133°}{c}$

$c = \frac{900\sin 133°}{\sin 12°} \approx 3165.86\text{ m}$

② $\sin 35° = \frac{b}{c}$ $\quad \sin 35° = \frac{b}{3165.86}$

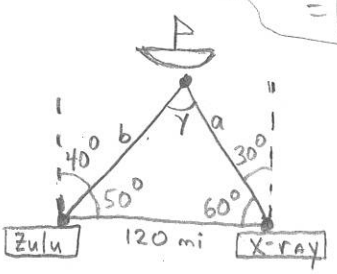
$b = 3165.86\sin 35°$

$b \approx 1816\text{ m}$

height of mountain $= 1816 + 2$

$= \boxed{1818\text{ m}}$

4. Coast Guard Zulu is located 120 miles due west of Station X-ray. A ship at sea sends an SOS call that is received by each station. The call to station Zulu indicates that the bearing of the ship from Zulu is N40°E. The call to Station X-ray indicates that the bearing of the ship from X-ray is N30°W.
   A. How far is each station from the ship?
   B. If a helicopter capable of flying 200 mph is dispatched from the station nearest to the ship, how long will it take to reach the ship?



< work is on the other side >

a.) ① $y = 180 - (50 + 60) = 70^\circ$

$$\frac{\sin 50^\circ}{a} = \frac{\sin 70^\circ}{120} \qquad \frac{\sin 60^\circ}{b} = \frac{\sin 70^\circ}{120}$$

$$a = \frac{120 \sin 50^\circ}{\sin 70^\circ} \approx \boxed{97.82 \text{ mi}} \qquad b = \frac{120 \sin 60^\circ}{\sin 70^\circ} \approx \boxed{110.59 \text{ mi}}$$

X-ray to the ship $\approx$ 97.82 mi Zulu to the ship $\approx$ 110.59 mi

b.) Distance = velocity · time

$$\frac{\text{Distance}}{\text{velocity}} = \text{time}$$

$$\frac{97.82 \text{ mi}}{200 \text{ mph}} = \text{time}$$

time $\approx$ 0.49 hrs $\approx$ $\boxed{29 \text{ minutes}}$

* It will take about 29 mins For the helicopter to reach the ship.